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The bang of a white hole in the early universe from a 6D vacuum state: origin of astrophysical spectrum

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Abstract. Using a previously introduced model in which the expansion of the universe is driven by a single scalar field subject to gravitational attraction induced by a white hole during the expansion (from a 6D vacuum state), we study the origin of squared inflaton fluctuations spectrum on astrophysical scales.

1 Introduction

In a previous work [1], we introduced a new formalism where, instead of implementing a dynamical foliation by taking a spatial dependence of the fifth coordinate including its time dependence, we considered another extra dimension, the sixth dimension, making possible the implementation of two dynamical foliations in a sequential manner. The first one was considered by choosing the fifth coordinate depending of the cosmic time, and the second one by choosing the sixth coordinate as dependent of the 3D spatial coordinates (in our case considered as isotropic). Of course, all of these choices preserve the continuity of the metric. In addition, the 6D metric must be Ricci-flat. This requirement is a natural extension of the vacuum condition used in the STM theory [2], in which 5D Ricci-flat metrics are used [3–8] and the cylinder condition has been eliminated in favor of retaining the metric's dependence on the extra coordinate. In simple words, we used the Campbell–Magaard theorem [9] and its extensions for embedding a 5D Ricci-flat space-time in a 6D Ricci-flat space-time. The conditions of 6D Ricci-flatness and the continuity of the metric give us the foliation of the sixth coordinate. These conditions specify the sixth dimension as a function of the 3D spatial coordinates, in order to establish the foliation. This function, for a particular 6D metric, can be seen in 4D as a gravitational potential related to a localized compact object that has the characteristics of a white hole. From a more general point of view, this is a mechanism for inducing localized matter onto a time-varying 4D hypersurface by establishing a spatial foliation of a sixth coordinate from a 6D Ricci-flat metric. The importance of this approach lies in that it can describe matter at both, cosmological and astro-

physical scales, in an expanding universe. The use of 6D physics is currently popular in particle physics (see, for example, [12]), but with the sixth dimension as time-like, rather than space-like.

In this letter we aim to study some predictions of this model on astrophysical scales. The power spectrum of matter is one of the most important statistics to describe the large-scale and astrophysical-scale structures of the universe. The studies developed in the last years have shown that on astrophysical scales the power spectrum of galaxies and clusters of galaxies can be satisfactorily expressed by a power law with an index between -1.9 and -1.5 [13]. On larger scales the spectrum turns over reaching a maximum on scales of $(100\text{–}150)h^{-1}\,\mathrm{Mpc}$. We shall suppose that the evolution of structure in the universe is only due to gravity.

2 Review of the formalism

2.1 Effective 4D dynamics from a 6D vacuum state

In order to describe a 6D vacuum, we consider the recently introduced 6D Riemann-flat metric [1]

$$\mathrm{d}S^2 = \psi^2\,\mathrm{d}N^2 - \psi^2 \mathrm{e}^{2N}[\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2] - \mathrm{d}\psi^2 - \mathrm{d}\sigma^2,\quad (1)$$

which defines a 6D vacuum state $G_{ab} = 0$ ($a, b = 0, 1, 2, 3, 4, 5$). We consider the 3D spatial space in spherical coordinates: $\mathbf{r} \equiv \mathbf{r}(r, \theta, \phi)$; here $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\,\mathrm{d}\phi^2$. The metric (1) resembles the 5D Ponce de Leon one [14], but with one additional space-like dimension. Furthermore, the coordinate N is dimensionless and the extra (space-like) coordinates ψ and σ are considered as noncompact. We define a physical vacuum state on the metric (1) through the

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action for a scalar field φ, which is nonminimally coupled to gravity:

$$I = \int d^6x \left[\frac{^{(6)}\mathcal{R}}{16\pi G} + \frac{1}{2} g^{ab} \varphi_{,a} \varphi_{,b} - \frac{\xi}{2} {}^{(6)}\mathcal{R} \varphi^2 \right], \quad (2)$$

where $^{(6)}\mathcal{R} = 0$ is the Ricci scalar and ξ gives the coupling of φ with gravity. Implementing the coordinate transformation $N = Ht$ and $R = r\psi = r/H$ on the frame $U^\psi = (d\psi/dS) = 0$ (considering H as a constant), followed by the foliation $\psi = H^{-1}$ on the metric (1), we obtain the effective 5D metric

$$^{(5)} dS^2 = dt^2 - e^{2Ht}[dR^2 + R^2 d\Omega^2] - d\sigma^2, \quad (3)$$

which is not Ricci-flat, because $^{(5)}\mathcal{R} = 12H^2$. However, it becomes Riemann-flat in the limit $H \to 0$ i.e. $R^A_{BCD}\big|_{H\to 0} = 0$, so that in this limit a 5D vacuum given by $G_{AB}\big|_{H\to 0} = 0$, $(A, B = 0, 1, 2, 3, 4)$. Hence, we can take the foliation $d\sigma^2 = 2\Phi_n(R) dR^2$ in the metric (3) on the sixth coordinate, and we obtain the effective 4D metric

$$^{(4)} dS^2 = dt^2 - e^{2Ht} \left[(1 + 2\Phi_n(R)) dR^2 + R^2 d\Omega^2 \right], \quad (4)$$

where t is the cosmic time, and $H = \dot{a}/a$ is the Hubble parameter for the scale factor $a(t) = a_0 e^{Ht}$, with $a_0 = a(t = 0)$. The Einstein equations for the effective 4D metric (4) are $G_{\mu\nu} = -8\pi G T_{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$, where $T_{\mu\nu}$ is represented by a perfect fluid: $T_{\mu\nu} = (p + \rho)u_\mu u_\nu - g_{\mu\nu} p$, p and ρ being the pressure and the energy density on the effective 4D metric (4). In a previous work [1] $\Phi_n(R)$ was found for a punctual mass $M_n = nM_P/3$ ($M_P = 1.2 \times 10^{19}$ GeV is the Planckian mass) located at $R = 0$, in the absence of expansion $(H \to 0)$:

$$\Phi_n(R) = \frac{-3GM_n \ln(R/R_*)}{R + 6GM_n \ln(R/R_*)}. \quad (5)$$

Here, R_* is the value of R such that $\Phi_n(R_*) = 0$ and $G = M_P^{-2}$ the gravitational constant. Hence, the function $\Phi_n(R)$ describes the geometrical deformation of the metric induced from a 5D flat metric, by a mass M_n. This function is $\Phi_n > 0$ (or $\Phi_n < 0$) for $R < R_*$ ($R > R_*$), respectively. Furthermore, $\Phi_n(R)\big|_{R\to\infty} \to 0$, and thereby the effective 4D metric (4) is (in their 3D ordinary spatial components) asymptotically flat. In this analysis we are considering the usual 4-velocities $u^\alpha = (1, 0, 0, 0)$. The equation of state for a given R is [1]

$$\frac{p}{\rho} = -1 - \frac{2GM_n}{R^3} \frac{[1 - \ln(R/R_*)]e^{-2Ht}}{[H^2 - \frac{2GM_n}{R^3} e^{-2Ht}]}, \quad (6)$$

p being $p = p_R + p_\theta + p_\phi$. From (6) we can see that at the end of inflation, when the number of e-folds is sufficiently large, the second term in (6) becomes negligible on cosmological scales (on the infrared (IR) sector), and the equation of state on this sector describes an asymptotic vacuum dominated (inflationary) expansion:

$$p\big|_{IR}^{(end)} \simeq -\rho\big|_{IR}^{(end)}. \quad (7)$$

On the other hand, for $t = 0$, we obtain (on arbitrary scales)

$$\frac{p}{\rho}\bigg|_{t=0} = -\frac{\left[H^2 - \frac{2GM_n}{R^3} \ln(R/R_*) \right]}{\left[H^2 - \frac{2GM_n}{R^3} \right]}, \quad (8)$$

which for a non-expanding universe $(H \to 0)$ gives us, for $R < R_*$, the equation of state for primordial galaxies,

$$\frac{p}{\rho}\bigg|_{H\to 0, t=0} \simeq \ln(R_*/R), \quad (9)$$

which means that primordial galaxy formation should be possible on scales smaller than R_*.

The effective 4D action for the universe is

$$^{(4)}I = \int d^4x \left[\frac{^{(4)}\mathcal{R}}{16\pi G} + \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - \frac{\xi_1}{2} {}^{(4)}\mathcal{R}\bigg|_{\Phi_n=0} \varphi^2 \right. $$
$$\left. - \frac{\xi_2^{n,l}(R)}{2} {}^{(4)}\mathcal{R}\bigg|_{H=0} e^{-2Ht} \varphi^2 \right], \quad (10)$$

where $^{(4)}\mathcal{R} = 12H^2 - (12GM_n/R^3)e^{-2Ht}$ is the effective 4D Ricci scalar for the effective 4D metric (4), ξ_1 gives the coupling of the scalar field $\varphi(\mathbf{R}, t)$ with gravity on the background induced by the foliation of the first extra dimension ψ at $\Phi_n(R) = 0$, and $\xi_2^{n,l}(R)$ gives the coupling of φ with gravity, on the background induced by the foliation of the second extra dimension σ at $H = 0$. The equation of motion for the field φ on the metric (4) is

$$\ddot{\varphi} + 3H\dot{\varphi} + e^{-2Ht} \left[\frac{1}{R^2(1 + 2\Phi_n(R))} \frac{\partial}{\partial R} \left[R^2 \frac{\partial\varphi}{\partial R} \right] \right.$$
$$- \frac{1}{(1 + 2\Phi_n(R))^2} \frac{\partial\Phi_n}{\partial R} \frac{\partial\varphi}{\partial R} + \frac{1}{R^2 \sin(\theta)} \frac{\partial}{\partial\theta} \left[\sin(\theta) \frac{\partial\varphi}{\partial\theta} \right]$$
$$\left. + \frac{1}{R^2 \sin^2(\theta)} \frac{\partial^2\varphi}{\partial\phi^2} \right]$$
$$+ \left[\xi_1 {}^{(4)}\mathcal{R}\bigg|_{\Phi_n=0} + \xi_2^{n,l}(R) {}^{(4)}\mathcal{R}\bigg|_{H=0} e^{-2Ht} \right] \varphi$$
$$= 0. \quad (11)$$

Notice that under this approach, the expansion is affected by a geometrical deformation induced by the gravitational attraction of a white hole of mass $M_n = nM_P/3$. This deformation is described by the function $\Phi_n(R)$, which tends to zero on cosmological scales [see Fig. 1].

2.2 Weak field approximation

In this section we study the weak field approximation for (11). In that limit approximation the function $\Phi_n(R)$ in (5) can be written as

$$\Phi_n(R) \simeq -\frac{3GM_n}{R} + \left(\frac{6GM_n}{R} \right)^2, \quad (12)$$

M_n being the mass of the compact object located at $R = 0$. Note that the function (12), as well as the exact one (5),
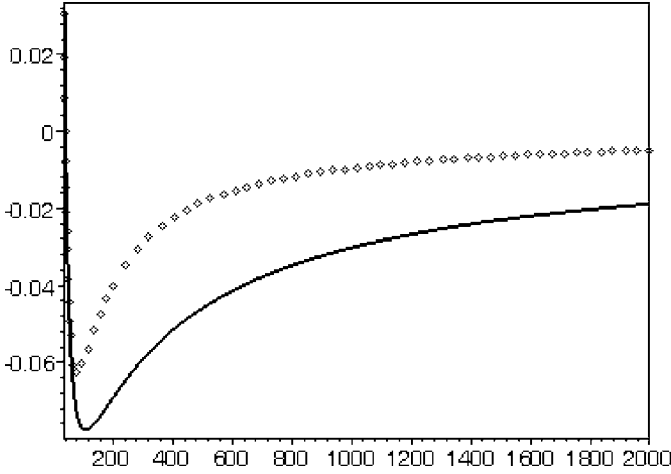
Fig. 1. We show the functions $\Phi_n(R)$ as a function of $0.9R_* < R < 50R_*$ ($R_* = 4n\lambda_P - \lambda_P$ is the Planckian wavelength) in their exact (*continuous line*) and approximated versions (*pointed line*), corresponding to (5) and (12), respectively. We use the values $n = 10$, $l = 0$ and $G = 1$ (only in the figure)

goes to zero at $R \to \infty$. For $M_n > 0$ there is a stable equilibrium for test particles at $R_* = 12GM_n$ and it exhibits a gravitational repulsion (antigravity) for $R < R_*$. Hence, this object has the properties of a white hole [15]. In Fig. 1 we have plotted $\Phi_n(R)$ given by (5) (the exact expression plotted with a continuous line) and (12) (the weak field approximated expression plotted with a pointed line), respectively, for $R > R_*$. Notice that the difference between both is more important on smaller scales. Furthermore, it is evident that the exact expression of $\Phi_n(R)$ is more sensitive to the interaction. In order to obtain solutions of (11) we propose $\varphi(\mathbf{R}, t) \sim \varphi_t(t)\varphi_R(R)\varphi_{\theta,\phi}(\theta, \phi)$. With this choice and using the fact that (see [1, (11)]): $-3GM_n = \frac{R^2 \, \mathrm{d}\Phi_n/\mathrm{d}R}{(1+2\Phi_n)^2} + \frac{R\Phi_n}{1+2\Phi_n}$, we obtain

$$\ddot{\varphi}_t + 3H\varphi_t - 12H^2\xi_1\varphi_t = -\alpha_t\varphi_t e^{-2Ht}, \tag{13}$$

$$\frac{\partial}{\partial R}\left[R^2\frac{\partial\varphi_R}{\partial R}\right] + [3GM_n(1+2\Phi_n) + R\Phi_n]\frac{\partial\varphi_R}{\partial R}$$
$$= \varphi_R\left[\frac{12GM_n}{R}\xi_2^{n,l}(R) - \alpha_t R^2 + \alpha_R\right](1+2\Phi_n), \tag{14}$$

$$\sin(\theta)\frac{\partial}{\partial\theta}\left[\sin(\theta)\frac{\partial\varphi_{\theta,\phi}}{\partial\theta}\right] + \frac{1}{\sin^2(\theta)}\frac{\partial^2\varphi_{\theta,\phi}}{\partial\phi^2} = \alpha_R\varphi_{\theta,\phi}, \tag{15}$$

where α_t and α_R are separation constants. The solution of (15) is given by the spherical harmonics $Y_{l,m}(\theta, \phi)$ $[\alpha_R = l(l+1)]$

$$\varphi_{\theta,\phi}(\theta, \phi) \sim \sum_{l,m} A_{lm}Y_{l,m}(\theta, \phi)$$
$$= \sum_{l,m} A_{lm}\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}\mathcal{P}_l^m(\cos(\theta)), \tag{16}$$

where $m = -l, -(l-1), \ldots, 0, \ldots, (l-1), l$ is a separation constant and $\mathcal{P}_l^m(\cos(\theta))$ are the Legendre polynomials: $P_l^m(x) = [(-1)^m/(2^l l!)](1-x^2)^{m/2}(\mathrm{d}^{l+m}/\mathrm{d}x^{l+m})(x^2-1)^l$.

In order to study astrophysical implications for the solutions of (13)–(15), we shall concentrate on the dispersive case ($\alpha_t = |\alpha_t| = k_R^2$) for $\varphi(\mathbf{R}, t)$. In this case k_R is the wavenumber related to the coordinate R. On the other hand, for solving (14), we propose

$$\varphi_R(R) = \varphi_{k_R}(R)e^{-\int f_n(R)\,\mathrm{d}R}, \tag{17}$$

such that $\varphi_{k_R}(R) = \varphi_R[\Phi_n = 0]$. With this choice (14) can be replaced by the equations

$$\frac{\mathrm{d}^2\varphi_{k_R}}{\mathrm{d}R^2} + \frac{2}{R}\frac{\mathrm{d}\varphi_{k_R}}{\mathrm{d}R} + \varphi_{k_R}\left[k_R^2 - \frac{l(l+1)}{R^2}\right] = 0, \tag{18}$$

$$\frac{\mathrm{d}\varphi_{k_R}}{\mathrm{d}R}[6GM_n(1+2\Phi_n) + 2R\Phi_n]$$
$$+ \varphi_{k_R}\left[R^2\left(f_n^2 - \frac{\mathrm{d}f_n}{\mathrm{d}R}\right) - 2Rf_n\right.$$
$$+ 2\Phi_n\left(k_R^2R^2 - l(l+1)\right) - 3GM_n(1+2\Phi_n)f_n$$
$$\left. - R\Phi_n f_n - \frac{12GM_n\xi_2^{n,l}(R)}{R}(1+2\Phi_n)\right]$$
$$= 0. \tag{19}$$

The solution for (18) is

$$\varphi_{k_R}(R) = \frac{A}{\sqrt{R}}\mathcal{J}_{l+1/2}[k_RR], \tag{20}$$

where $\mathcal{J}_{l+1/2}$ are the Bessel functions. On the other hand, from (19) we obtain the coupling $\xi_2^{n,l}(R)$,

$$\xi_2^{n,l}(R) = \frac{R}{12GM_n(1+2\Phi_n)}$$
$$\times\left\{\frac{1}{\varphi_{k_R}}\frac{\mathrm{d}\varphi_{k_R}}{\mathrm{d}R}[6GM_n(1+2\Phi_n) + 2R\Phi_n]\right.$$
$$- \left[R^2\left(f_n^2 - \frac{\mathrm{d}f_n}{\mathrm{d}R}\right) - 2Rf_n\right.$$
$$+ 2\Phi_n\left[k_R^2R^2 - l(l+1)\right]$$
$$\left.\left. - 3GM_n(1+2\Phi_n)f_n - R\Phi_n f_n\right]\right\}, \tag{21}$$

with $f_n(R) = -[1/(2R^2)][3GM_n(1+2\Phi_n(R)) + R\Phi_n(R)]$. The solution of (13) for the dispersive case is

$$\varphi_t(t) \sim \xi_{k_R}(t) = Ae^{-3Ht/2}\mathcal{H}_\nu^{(2)}\left[\frac{k_R}{H}e^{-Ht}\right], \tag{22}$$

A being a constant, $\nu = (1/2)\sqrt{9 + 48\xi_1}$ and $\mathcal{H}_\nu^{(2)}$ is a Hankel function of the second kind. The complete solution for the field $\varphi(\mathbf{R}, t)$ can be written as

$$
\varphi(\mathbf{R}, t) = \frac{1}{(2\pi)^{3/2}} e^{-\left[\left(\frac{n\lambda_P}{2R}\right)^2 \left(1 + \frac{16n\lambda_P}{3R}\right)\right]}
$$
$$
\times \int d^3 k_R \sum_{m=-l}^{l} \sum_{l=0}^{n-1} \left[a_{k_R l m} Y_{l,m}(\theta, \phi) \varphi_{k_R}(R) \xi_{k_R}(t) \right.
$$
$$
\left. + a_{k_R l m}^\dagger Y_{l,m}^*(\theta, \phi) \varphi_{k_R}^*(R) \xi_{k_R}^*(t) \right], \tag{23}
$$

where $\varphi_{k_R}(R)$ and $\xi_{k_R}(t)$ are given respectively by the expressions (20) and (22).

3 Astrophysical-scale spectrum

In this section we shall study the structure formation during inflation on astrophysical scales. In the framework of cosmological scales, we understand that these are small scales, on which one feels the presence of the compact object (and thus we shall consider $\Phi_n(R) \neq 0$). We must understand the present astrophysical scales (~ 100 Mpc). We shall refer to this part of the spectrum as the small-scale (SS) spectrum. It is known from observation that galaxies and clusters of galaxies are correlated. This should be responsible for differences in the clustering properties of the populations in the nearby universe. One population is characteristic for rich superclusters, and the other for poorer ones. The former population has a power spectrum with a sharp peak and a correlation function with zero crossing near $60h^{-1}$ Mpc. The later population has a flatter power spectrum and a zero crossing of the correlation function near $40h^{-1}$ Mpc.

The effective small-scale squared fluctuations in presence of $\Phi_n(R) \neq 0$ are given by [1]

$$
\langle \varphi^2 \rangle_{\text{SS}}^{(\Phi_n(R) \neq 0)} = \frac{1}{2\pi^2} e^{-\left[\frac{1}{2}\left(\frac{n\lambda_P}{R}\right)^2 \left(1 + \frac{16n\lambda_P}{3R}\right)\right]}
$$
$$
\times \int_{\epsilon k_H}^{k_H} \frac{d k_R}{k_R} k_R^3 \left[\xi_{k_R} \xi_{k_R}^* \right] \Big|_{\text{IR}}^{(\Phi_n(R) \simeq 0)}, \tag{24}
$$

where $\epsilon = (k_{\max}^{(\text{IR})}/k_P) \ll 1$ is a dimensionless constant parameter, and $k_{\max}^{\text{IR}} = k_H(t_i) = \left.\sqrt{12\xi_1 + (9/4)} H e^{Ht}\right|_{t=t_i}$ is the wavenumber related with the Hubble radius at the time t_i (when the horizon re-enters), and k_P is the Planckian wavenumber. The asymptotic expression for the modes on the IR sector is [1]

$$
\xi_{k_R}(t)\Big|_{\text{IR}}^{(\Phi_n(R) \simeq 0)} \simeq -\frac{i}{2}\sqrt{\frac{1}{\pi H}} \Gamma(\nu) \left(\frac{k_{\bar{R}}}{2H}\right)^{-\nu} e^{Ht(\nu - 3/2)}. \tag{25}
$$

Inserting (25) into (24) we obtain

$$
\langle \varphi^2 \rangle_{\text{SS}}^{(\Phi_n(R) \neq 0)} = \frac{2^{2\nu-3}}{3 - 2\nu} \frac{\Gamma^2(\nu)}{\pi^3} H^{2\nu-1} e^{-(3-2\nu)Ht}
$$
$$
\times e^{-\left[\frac{1}{2}\left(\frac{n\lambda_P}{R}\right)^2 \left(1 + \frac{16n\lambda_P}{3R}\right)\right]} k_H^{3-2\nu} (1 - \epsilon)^{3-2\nu}. \tag{26}
$$

Hence, the expression (26) becomes

$$
\langle \varphi^2 \rangle_{\text{SS}}^{(\Phi_n(R) \neq 0)} \simeq e^{-\left[\frac{1}{2}\left(\frac{n\lambda_P}{R}\right)^2 \left(1 + \frac{16n\lambda_P}{3R}\right)\right]} \langle \varphi^2 \rangle_{\text{IR}}^{(\Phi_n(R) \simeq 0)}, \tag{27}
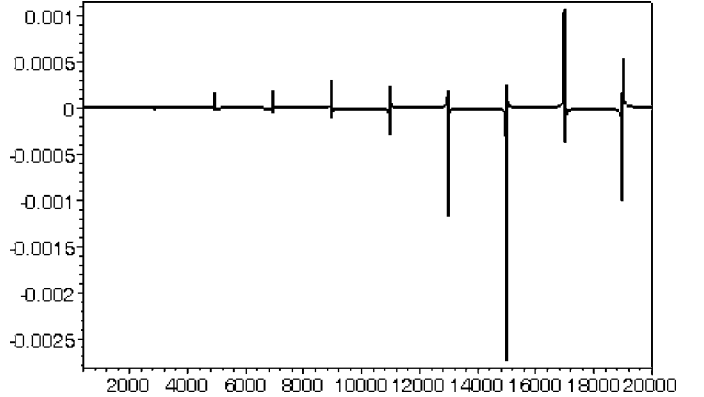$$

Fig. 2. The figure shows the coupling $\xi_2^{n,l}(R)$ as a function of $10R_* < R < 500R_*$ ($R_* = 4n\lambda_P - \lambda_P$ is the Planckian wavelength). We use the values $n = 10$, $l = 0$ and $G = 1$ (only in the figure)
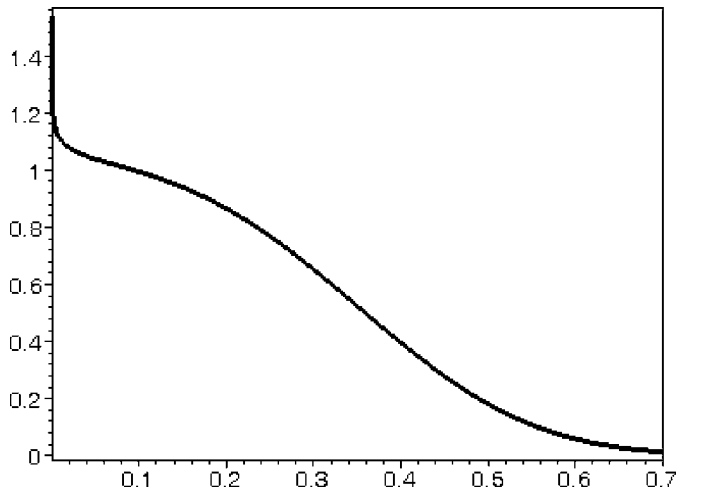

Fig. 3. The figure shows the power spectrum $\mathcal{P}(k_R)$ of $\langle \varphi^2 \rangle_{\text{SS}}$ as a function of $0.0001 < k_R < 0.7$ (we take the Planckian wavenumber value as $k_P = 2\pi$). We use the values $n = 10$, $l = 0$, $G = 1$ (only in the figure) and $n_s = 0.964$ (which corresponds to $\nu = 1.518$)

where [1]

$$\langle \varphi^2 \rangle_{\mathrm{IR}}^{(\Phi_n(R) \simeq 0)} = \frac{2^{2\nu-3}}{3-2\nu} \frac{\Gamma^2(\nu)}{\pi^3} H^{2\nu-1} e^{-(3-2\nu)Ht} k_H^{3-2\nu}$$

$$= \frac{2^{2\nu-3} H^2 \Gamma^2(\nu)}{\pi^3 (3-2\nu) \left(12\xi_1 + \frac{9}{4}\right)^{\nu-3/2}} . \qquad (28)$$

Finally, we can write the power spectrum of $\langle \varphi^2 \rangle_{\mathrm{SS}}$, making $k_R = 2\pi/R$ and $k_{\mathrm{P}} = 2\pi/\lambda_{\mathrm{P}}$,

$$\mathcal{P}(k_R)|_{\langle \varphi^2 \rangle_{\mathrm{SS}}} \sim e^{-\left[\frac{1}{2}\left(n\frac{k_R}{k_{\mathrm{P}}}\right)^2 \left(1 + \frac{16n}{3}\frac{k_R}{k_{\mathrm{P}}}\right)\right]} k_R^{3-2\nu} . \qquad (29)$$

In Fig. 2 we have plotted the coupling parameter $\xi_2^{n,l}(R)$ in (21) on astrophysical scales ($R_* < R < 500R_*$), for $n = 10$, $l = 0$ and $A = 10^{12}$. Notice that we have used $\varphi_{k_R}(R)$ given in (20) and $\Phi_n(R)$ given in (12). It is evident that the coupling parameter becomes more important as R increases (i.e., for bigger scales). In Fig. 3 we show the power spectrum $\mathcal{P}(k_R)|_{\langle \varphi^2 \rangle_{\mathrm{SS}}}$ as a function of the wavenumber k_R on astrophysical scales. There are three indices that are important on the whole spectrum. Two of these dominate on astrophysical scales. The first one for $k_R > 0.4$, the second one on the range $0.01 < k_R < 0.4$. The third index $n_s \simeq 1$ dominates on cosmological scales ($k_R \ll 0.01$). We have used $n_s = 0.964$ (which corresponds to $\nu = 1.518$) in all the graphics.

4 Final comments

We have studied the power spectrum on astrophysical scales of the φ^2-expectation value. The model here studied predicts a spectrum that agrees qualitatively with experimental data [16]. An interesting result is that we detect two different sectors with different power indices, which dominate on different astrophysical scales. The third index is n_s, which is relevant on cosmological scales. Another interesting result is the periodicity of the coupling $\xi_2^{n,l}(R)$. Notice that its amplitude increases with R. This coupling should be responsible for the correlation of the galaxies on astrophysical scales.

For simplicity, in this letter we considered a de Sitter expansion, where the Hubble parameter H is a constant. However, the formalism could be extended to whatever $H = H(t)$. The possibility of having a dynamical foliation using only 5D (in models without gravitational sources), was explored [17]. In our model the fifth dimension is responsible for the 4D de Sitter expansion, which is physically driven by the inflaton field φ. From the physical point of view, the sixth dimension is responsible for the spatial curvature induced by the mass of the white hole (located at $R = 0$). In more general terms, the fifth dimension is physically related to the vacuum energy density, which is the source of the effective 4D global inflationary expansion, whereas the sixth one induces local gravitational sources.

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